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Thermoelectric spin transfer in textured magnets

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We study charge and energy transport in a quasi-one-dimensional magnetic wire in the presence of magnetic textures. The energy flows can be expressed in a fashion similar to charge currents, leading to energy-current-induced spin torques. In analogy to charge currents, we can identify two reciprocal effects: spin torque on the magnetic order parameter induced by energy current and the Berry-phase gauge-field-induced energy flow. In addition, we phenomenologically introduce β-like viscous coupling between magnetic dynamics and energy current into the Landau-Lifshitz-Gilbert equation, which originates from spin mistracking of the magnetic order. We conclude that the introduced term is important for the thermally induced domain-wall motion. We study the interplay between charge and energy currents and find that many of the effects of texture motion on the charge currents can be replicated with respect to energy currents. For example, the moving texture can lead to energy flows which is an analog of the electromotive force in case of charge currents. We suggest a realization of cooling effect by magnetic texture dynamics.

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The notion of the Berry phase[1] naturally appears in the description of magnetic texture dynamics in the limit of strong exchange field. Electrons with spin up and down with respect to the local magnetization experience fictitious electromagnetic fields of opposite sign.[2] It has been realized that the spin-transfer torque (STT) is a reciprocal effect to the electromotive force (EMF) associated with this Lorentz force.[3,4] In real systems, the exchange field is finite leading to spin misalignments with the texture, and more realistic description should take such effects into account via the so-called β terms in the Landau-Lifshitz-Gilbert equation (LLGE).[5,6]

The interest in magneto-thermoelectric effects has recently surged as experimental data has been available.[7] The Peltier effect describes heat transfer accompanying the current flow. The opposite is the Seebeck effect that describes the thermo-EMF induced by temperature gradients. The Peltier and Seebeck thermoelectric effects as well as the thermoelectric STT's have been studied in multilayered nanostructures.[8] Hatami *et al.* proposed a thermoelectric STT as mechanism for domain-wall motion (DWM).[8] Berger *et al.* observed and discussed DWM induced by heat currents.[9] Thermal STT's may soon be employed in the next-generation nonvolatile data elements for reversal of magnetization. Thermoelectric nanocoolers can find applications in the nanoelectronic circuits and devices.[10]

In this Rapid Communication, we study continuous magnetic systems relevant to DWM (Ref. 11) and spin-textured magnets.[12] We phenomenologically describe thermal STT's in a quasi-one-dimensional (1D) magnetic wire with magnetic texture. The Berry-phase gauge-field-induced energy flow turns out to be reciprocal effect to the thermal STT and both effects can be formally eliminated from the equations of motion by properly redefining the thermodynamic variables, which is reminiscent of the nondissipative STT's.[4] We further generalize our description by including viscous terms corresponding to spin mistracking. These viscous effects turn out to be important for the thermally induced DWM and can lead to effects such as cooling by magnetic texture dynamics. We also find that the Peltier and Seebeck effects can be modified and tuned by the magnetic texture dynamics.

Consider a thin quasi-1D magnetic wire with the local direction of spin density $\mathbf{m}(x,t)$ in the presence of chemical potential $\mu(x,t)$ and temperature $T(x,t)$ gradients. We would like to construct a phenomenological description of our system based on thermodynamic variables introduced above and their conjugate forces. The ferromagnetic wire is thermally isolated, and after being perturbed by nonequilibrium chemical potential, magnetization (Fig. 1) and temperature gradients, evolves back toward equilibrium according to the equations of motion, producing entropy. We allow this equilibrium state to be topologically nontrivial, e.g., a magnetic DW or vortex. The state of partial equilibrium can be described by thermodynamic variables x_i and their conjugates (generalized forces) $X_i = \partial \mathbb{S} / \partial x_i$ with the entropy and its time derivative being

$$\mathbb{S} = \mathbb{S}_0 - \frac{1}{2}\sum_{i,k=1}^{n} \beta_{ik} x_i x_k, \quad \dot{\mathbb{S}} = \sum_{i=1}^{n} X_i \dot{x}_i. \tag{1}$$

We initially consider the entropy $\mathbb{S}(\rho, \rho_U, \mathbf{m})$ as a function of the density of electron charge ρ, the density of energy ρ_U and the magnetization direction. The magnitude of the magnetization is not treated as a dynamic variable, assuming sufficiently fast spin-flip relaxation. The conservation laws of energy/charge provide linear relations,

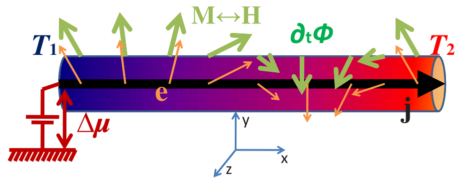

FIG. 1. (Color online) In quasi-1D magnetic wire, charge current density j is induced by potential gradients $\partial_x \mu$, temperature gradients $\partial_x T$, and EMF $\partial_t \Phi$ produced by the Berry phase Φ, which is acquired by the electron spin following the time-dependent magnetic profile. Coupled viscous processes arise once we relax the projection approximation. The magnetic texture $\mathbf{m}(x,t)$ responds to the effective field $\mathbf{H}(x,t)$.

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$$\dot{\rho}=-\partial_x j, \quad \dot{\rho}_U=-\partial_x j_U, \tag{2}$$

where we introduced the charge current j and energy current j_U. For conserved quantities, it is more convenient to work with fluxes j and j_U instead of densities ρ and ρ_U arriving at equivalent description due to the linear relations in Eq. (2).

We now focus on identifying the thermodynamic variables and their conjugates by calculating the time derivative of the entropy. Under the fixed texture, the rate of the entropy change is[13]

$$\dot{\mathbb{S}}=-\oint dx\frac{\partial_x j_U+\mu\dot{\rho}}{T}=-\oint dx\frac{\partial_x j_q+j\partial_x\mu}{T}, \tag{3}$$

where we introduced the modified energy current $j_q=j_U-\mu j$ that describes the energy flow offset by the energy corresponding to the chemical potential μ (hereafter, only the energy current j_q is used). The chemical potential is defined as the conjugate of the charge density. The introduction of j_q is necessary to avoid the unphysical gauge dependence of the energy current and the associated kinetic coefficients on the potential offset for the entire system. We can now write the rate of the entropy change for the general case of dynamic spin texture

$$\dot{\mathbb{S}}=\oint dx\left[j_q\partial_x\left(\frac{1}{T}\right)-j\frac{\partial_x\mu}{T}-\dot{\mathbf{m}}\cdot\frac{\mathbf{H}}{T}\right], \tag{4}$$

where in Eq. (3) we integrated the term involving j_q by parts and the conjugate/force corresponding to the magnetization is defined by the functional derivative $\delta_{\mathbf{m}}\mathbb{S}|_{Q,q}=-\mathbf{H}/T$ with $Q(x)$ and $q(x)$ being the overall charge and energy that passed the cross section at point x which corresponds to integrating j and j_q in time, respectively. As can be seen from Eq. (4), our other conjugates are $\delta_q\mathbb{S}|_{\mathbf{m},Q}=\partial_x(1/T)$ and $\delta_Q\mathbb{S}|_{\mathbf{m},q}=-\partial_x\mu/T$. In general, $\mathbf{H}$ is not the usual "effective field" corresponding to the variation of the Landau free-energy functional $F[\mathbf{m},\mu,T]$ and only when $\partial_x T=0$ and $\partial_x\mu=0$ the "effective fields" coincide. Let us initially assume that even in an out-of-equilibrium situation, when $\partial_x T\neq 0$ and $\partial_x\mu\neq 0$, $\mathbf{H}$ depends only on the instantaneous texture $\mathbf{m}(x)$. In general, however, we may expand $\mathbf{H}$ phenomenologically in terms of small $\partial_x T$ and $\partial_x\mu$.

In our phenomenological theory, the time derivatives of thermodynamic variables are related to the thermodynamic conjugates via the kinetic coefficients. In order to identify the kinetic coefficients, we assume that the currents j and j_q are determined by the chemical potential and temperature gradients as well as the magnetic wire dynamics, which exerts fictitious Berry-phase gauge fields[4] on the charge transport along the wire. We then have for the charge/energy-current gradient expansion

$$j=-\tilde{g}\partial_x\mu+\tilde{\xi}\frac{\partial_x T}{T}+\tilde{p}(\mathbf{m}\times\partial_x\mathbf{m}+\tilde{\beta}\partial_x\mathbf{m})\cdot\dot{\mathbf{m}}, \tag{5}$$

$$j_q=\tilde{\xi}\partial_x\mu-\tilde{\zeta}\partial_x T+\tilde{p}'(\mathbf{m}\times\partial_x\mathbf{m}+\tilde{\beta}'\partial_x\mathbf{m})\cdot\dot{\mathbf{m}}, \tag{6}$$

where we assume that the coefficients $\tilde{g}$, $\tilde{\xi}$, $\tilde{\zeta}$ can in general also depend on temperature and texture, for the latter, to the leading order, as $\tilde{g}=\tilde{g}_0+\eta_{\tilde{g}}(\partial_x\mathbf{m})^2$, etc. In Eqs. (5) and (6), we expand only up to the linear order in the nonequilibrium quantities $\partial_x\mu$, $\partial_x T$, $\dot{\mathbf{m}}$ and to the second order in $\partial_x\mathbf{m}$; the latter terms are expected to be small in practice and only are necessary for establishing the positive definiteness of the response matrix. The spin-rotational symmetry of the magnetic texture and the inversion symmetry of the wire are also assumed to avoid additional and often complicated terms in our expressions. Relating $\dot{\mathbf{m}}$ to the generalized force $-\mathbf{H}/T$, within the LLG (Ref. 14) phenomenology, we derive the modified LLGE consistent with Eqs. (5) and (6), with the guidance of the Onsager reciprocity principle,

$$\begin{aligned}s(1+\alpha\mathbf{m}\times)\dot{\mathbf{m}}+\mathbf{m}\times\mathbf{H}=&-\tilde{p}(\partial_x\mathbf{m}+\tilde{\beta}\mathbf{m}\times\partial_x\mathbf{m})\partial_x\mu\\&-\tilde{p}'(\partial_x\mathbf{m}+\tilde{\beta}'\mathbf{m}\times\partial_x\mathbf{m})\partial_x T/T,\end{aligned} \tag{7}$$

where we introduced the spin density s so that $s\mathbf{m}=\mathbf{M}/\gamma$, with M being the magnetization density and γ the gyromagnetic ratio ($\gamma<0$ for electrons); Eq. (7) can be written in terms of the charge/energy flows by inverting the linear relation $\{j,j_q\}=\{\tilde{g},\tilde{\xi};\tilde{\xi},\tilde{\zeta}\}\{\partial_x\mu,\partial_x T/T\}$,

$$\partial_x\mu=-gj+\xi j_q+p(\mathbf{m}\times\partial_x\mathbf{m}+\beta\partial_x\mathbf{m})\cdot\dot{\mathbf{m}}, \tag{8}$$

$$\partial_x T/T=\xi j-\zeta j_q+p'(\mathbf{m}\times\partial_x\mathbf{m}+\beta'\partial_x\mathbf{m})\cdot\dot{\mathbf{m}}, \tag{9}$$

$$\begin{aligned}s(1+\alpha\mathbf{m}\times)\dot{\mathbf{m}}+\mathbf{m}\times\mathbf{H}=&\,p(\partial_x\mathbf{m}+\beta\mathbf{m}\times\partial_x\mathbf{m})j\\&+p'(\partial_x\mathbf{m}+\beta'\mathbf{m}\times\partial_x\mathbf{m})j_q,\end{aligned} \tag{10}$$

where the coefficients g, ξ, ζ, p, p', β, and β' can be expressed via the ones with tilde. In Eq. (10), we disregarded corrections of order $\sim(\partial_x\mathbf{m})^2\dot{\mathbf{m}}$ to the Gilbert damping, noting that terms at this order can emerge after solving the above coupled equations.[4] The kinetic coefficients contain information about the conductivity, $\sigma=\tilde{g}$, the thermal conductivity, $\kappa=1/\zeta T$, and the conventional Seebeck and Peltier coefficients can be found from Eqs. (5) and (9) by assuming $j=0$ for the former, $S=-\tilde{\xi}/\tilde{g}T$, and by assuming $\partial_x T=0$ for the latter, $\Pi=\xi/\zeta=-\tilde{\xi}/\tilde{g}$, and $g=1/\sigma+S^2T/\kappa$. Equation (10) differs from an ordinary LLGE (Ref. 6) by the extra spin torque terms that appear in the presence of the energy flow j_q. These torques are similar to the nondissipative and dissipative current-induced spin torques:[4] the former (latter) can be related to electron spins following (mistracking) the magnetic texture. The phenomenological parameter $\tilde{p}$ (or $p=\tilde{p}/\sigma_0-p'\Pi_0$) can be approximated as $\tilde{p}/\sigma_0=\wp\hbar/2e$ in the strong exchange limit[4] and corresponds to the electron spin-charge conversion factor $\hbar/2e$ multiplied by the polarization $\wp=(\sigma_0^{\uparrow}-\sigma_0^{\downarrow})/\sigma_0$, $\sigma_0=\sigma_0^{\uparrow}+\sigma_0^{\downarrow}$, and $-e$ is the charge of particles ($e>0$ for electrons).

Similarly, we can consider Eq. (10) under the conditions of vanishing charge currents and fixed texture, and find the spin current resulting from the temperature gradients: $2eS_s\partial_x T/(1/\sigma_0^{\uparrow}+1/\sigma_0^{\downarrow})$ where $S_s=(S_0^{\uparrow}-S_0^{\downarrow})/e$ is the spin Seebeck coefficient. By involving the electron spin-charge conversion factor again, we can approximate the second spin torque parameter p' in Eq. (10) in the strong exchange limit arriving at

$$p' = -\frac{\hbar}{2e}\wp_S S_0 \frac{\sigma_0(1-\wp^2)}{\kappa_0}, \quad p = \frac{\wp\hbar}{2e} - p'\Pi_0, \tag{11}$$

where we introduced the spin polarization of the Seebeck coefficient $\wp_S=(S_0^{\uparrow}-S_0^{\downarrow})/(S_0^{\uparrow}+S_0^{\downarrow})=eS_s/2S_0$. The parameter β' describes misalignments of spins composing the thermally induced spin current and thus β' should be determined (up to some prefactor) by the ratio $\hbar/\tau_s\Delta_{xc}$ where τ_s is the spin-dephasing time and Δ_{xc} is the exchange splitting. Applicable to many metals, the Wiedemann-Franz law, according to which $\kappa_0/\sigma_0=LT$, where the Lorenz number $L=\pi^2k_B^2/3e^2$, allows to simplify Eq. (11). Effects such as spin drag[15] can influence the estimate in Eq. (11). The result in Eq. (11) can also be obtained from Eq. (9) by considering the texture-dynamics-induced EMF which can lead to the energy currents in the absence of charge currents.

One can define a magnetization variable,

$$\dot{\tilde{\mathbf{m}}} = \dot{\mathbf{m}} - pj\frac{1+\alpha\mathbf{m}\times}{(1+\alpha^2)s}\partial_x\mathbf{m} - p'j_q\frac{1+\alpha\mathbf{m}\times}{(1+\alpha^2)s}\partial_x\mathbf{m}, \tag{12}$$

which leads to the following changes in generalized forces:

$$T\delta_Q\mathrm{S}|_{\tilde{\mathbf{m}},q} = -\partial_x\mu + p(\mathbf{m}\times\partial_x\mathbf{m})\cdot\dot{\mathbf{m}},$$

$$T\delta_q\mathrm{S}|_{\tilde{\mathbf{m}},Q} = -\partial_xT/T + p'(\mathbf{m}\times\partial_x\mathbf{m})\cdot\dot{\mathbf{m}}. \tag{13}$$

This choice of variables absorbs the nondissipative (i.e., non β) texture-driven forces in Eqs. (5), (6), (8), and (9). The corresponding spin torques in Eqs. (7) and (10) will not be eliminated completely with a small correction remaining at order α^2.

From Eq. (4), we can write the rate of the entropy production,

$$\dot{\mathrm{S}} = \oint\frac{dx}{T}(gj^2+\zeta j_q^2-2\xi jj_q+\alpha s\dot{\mathbf{m}}^2-2\beta pj\dot{\mathbf{m}}\cdot\partial_x\mathbf{m} - 2\beta'p'j_q\dot{\mathbf{m}}\cdot\partial_x\mathbf{m}), \tag{14}$$

where $g=g_0+\eta_g(\partial_x\mathbf{m})^2$, $\xi=\xi_0+\eta_\xi(\partial_x\mathbf{m})^2$, and $\zeta=\zeta_0+\eta_\zeta(\partial_x\mathbf{m})^2$. The STT's in Eq. (10) induced by the charge/energy currents can be separated into the nondissipative and dissipative parts (β terms) based on Eq. (14). This separation is, nevertheless, formal as in realistic metallic systems the torques will always be accompanied by the dissipation due to the finite conductivities κ and σ. The dissipation in Eq. (14) is guaranteed to be positive definite if the following inequalities hold:

$$\xi\le\sqrt{g\zeta}, \quad \eta_g\ge\beta^2p^2/\alpha s, \quad \eta_\zeta\ge\beta'^2p'^2/\alpha s,$$

$$(\eta_\xi-\beta p\beta'p'/\alpha s)^2\le(\eta_g-\beta^2p^2/\alpha s)(\eta_\zeta-\beta'^2p'^2/\alpha s),$$

where the first inequality can be rewritten equivalently as $g\ge S^2T/\kappa$, and should always hold. Other inequalities are somewhat formal since their proof implies that our theory can describe sufficiently sharp and fast texture dynamics for dominating dissipation as opposed to the first inequality for proof of which a mere static texture assumption is sufficient. After replacing with equalities, the above inequalities can serve for crude estimates of the spin-texture resistivities (η_ζ,η_g) and the spin-texture Seebeck effect (η_ξ) due to spin dephasing. The condition on the spin-texture resistivity[4] $\eta_{1/\sigma}\ge(\tilde{\beta}\tilde{p})^2/\alpha s\sigma_0^2$ follows from the above inequalities.

Let us now discuss the thermal effects arising from the presence of magnetic texture. For a static spin texture and stationary charge density, $\partial_xj=0$, we can write the modification to the Thomson effect by calculating the rate of heat generation[13] $\dot{\mathrm{Q}}=-\partial_xj_U$ from Eqs. (5) and (9),

$$\dot{\mathrm{Q}} = \kappa\partial_x^2T+(\partial_T\kappa)(\partial_xT)^2+j^2/\sigma-T(\partial_TS)j\partial_xT + \eta_\kappa[\partial_x(\partial_x\mathbf{m})^2]\partial_xT - T\eta_S[\partial_x(\partial_x\mathbf{m})^2]j, \tag{15}$$

where $\kappa=\kappa_0+\eta_\kappa(\partial_x\mathbf{m})^2$ and $S=S_0+\eta_S(\partial_x\mathbf{m})^2$. The Thomson effect (i.e., the last term on the first line of the equation) can be modified by the presence of texture and some analog of local cooling may be possible even without temperature gradients (see the last term on the second line of the equation). The magnitude of the coefficients η_κ and η_S is not accessible at the moment and should be extracted from the microscopic calculations.

Another thermal effect we will discuss is related to heat flows induced by magnetization dynamics. As can be seen from Eq. (9), such heat flows can appear even in the absence of temperature gradients and charge current flows. When the magnetic texture follows a periodic motion, the energy flows should result in effective cooling or heating of some regions by specifically engineering the magnetic state of the wire and applied rf magnetic fields. The spin spring magnets can be of relevance.[16] Alternatively, the texture in our wire (i.e., spiral), can result from the Dzyaloshinskii-Moriya (DM) interaction[13] relevant for such materials as MnSi, (Fe,Co)Si or FeGe. In this case, the end of the spiral can be exchange coupled to a homogeneous magnetization of a magnetic film subject to rf magnetic field which should result in the rotation of magnetization in the film and spiral.

To simulate the spiral rotation and obtain the preliminary estimates of the effect, we consider the current-induced spiral motion which in turn leads to the energy flows due to the magnetic texture dynamics. The static texture corrections due to η-type terms will be ignored. We consider a ferromagnetic wire with the DM interaction in the absence of the temperature gradients. The "effective field" can be found from the free energy $F=\int d^3\mathbf{r}\mathcal{F}$ (Ref. 13):

$$\mathcal{F} = (J/2)[(\partial_x\mathbf{m})^2+(\partial_y\mathbf{m})^2+(\partial_z\mathbf{m})^2]+\Gamma\mathbf{m}\cdot(\nabla\times\mathbf{m}),$$

where J is the exchange coupling constant and Γ is the strength of the DM interaction. The "effective field" $\mathbf{H}\equiv\partial_{\mathbf{m}}F$ can be used in Eq. (10). The ground state of the Free energy F is a spiral state $\mathbf{m}(\mathbf{r})=\mathbf{n}_1\cos\mathbf{k}\cdot\mathbf{r}+\mathbf{n}_2\sin\mathbf{k}\cdot\mathbf{r}$ where the wavevector $\mathbf{k}=\mathbf{n}_3\Gamma/J$, and $\mathbf{n}_i$ form the right-handed orthonormal vector sets. We assume that the wavevector is along the wire, e.g., due to anisotropies. As can be found from LLG Eq. (7), the spiral starts to move along the wire in the presence of currents, according to[17]

$$\mathbf{m}(\mathbf{r},t) = m_x\mathbf{x} + m_\perp(\mathbf{y}\cos[k(x-\upsilon t)] + \mathbf{z}\sin[k(x-\upsilon t)]),$$

where the x axis points along the wire axis, $m_x=(j\hbar/e)(\beta/\alpha-1)/(2\Gamma-Jk)$, $m_\perp=\sqrt{1-m_x^2}$ and $\upsilon=\wp j(\tilde{\beta}/\alpha)\hbar/2es$. The wave number $k=2\pi/\lambda$ and m_x have been calculated numerically in Ref. 17 and in the presence of currents λ increases and m_x acquires some finite value; however, for an estimate corresponding to moderate currents the values given by the static spiral $k_0=\Gamma/J$ should suffice. The maximum current that the spiral can sustain without breaking into the chaotic motion is $j_{\max}\sim 2\Gamma e/\hbar$.[17]

Using Eq. (9), we find the energy flow accompanying the current flow as the spiral moves with the speed υ in the absence of the temperature gradients: $j_q=\Pi_0 j-m_\perp^2 k^2\upsilon p'\beta'/\zeta\approx 0.8\Pi_0 j$, where only β' term contributes to the energy flow in Eq. (9). We take parameters corresponding to MnSi (Ref. 12): the lattice constant $a=0.5$ nm, $M=0.4\mu_B/a^3$, $\lambda=20$ nm, $\sigma=50$ $(\mu\Omega\ \mathrm{m})^{-1}$, and $Ja=0.02$ eV. The typical Gilbert damping coefficients α, $\tilde{\beta}$ for transition-metal-based magnets are[6] $10^{-3}\sim 10^{-1}$ and the polarization $\wp=0.3\sim 1$, and although β' and $\wp_S$ can in general be different, in the estimates we will set $\beta'=\alpha=\tilde{\beta}=0.1$ and $\wp_S=\wp=0.8$ given the similarities between the mechanisms of $\beta'(\wp_S)$ and $\tilde{\beta}(\wp)$. By increasing $\tilde{\beta}$, β', σ and diminishing λ, α the energy flow can be made larger. We conclude then that the current-induced magnetic texture dynamics can lead to additional energy flows that in some cases can be comparable to the energy flows due to the Peltier effect. The renormalization of the Peltier coefficient also applies to the Seebeck coefficient due to the Onsager principle dictating that $S=\Pi/T$. In the absence of the temperature gradient, from Eq. (5), we also find correction to the conductivity caused by the EMF due to the spiral motion: $-\partial_x\mu=j/\sigma_0+m_\perp^2k^2\upsilon\wp\tilde{\beta}\hbar/2e\approx 0.8j/\sigma_0$. Repeating the estimates of the Peltier coefficient and conductivity for a Py wire,[11] we obtain 3 orders of magnitude smaller effects.

Finally, we calculate the speed of the spiral motion induced by temperature gradients in the absence of charge currents. In full analogy to the spiral motion induced by the charge currents and using Eq. (10), we find the spiral speed induced by the temperature gradients,

$$\upsilon=-p'\kappa\partial_x T\frac{\beta'}{\alpha}\frac{\hbar}{2es}=\wp_S S\partial_x T\sigma(1-\wp^2)\frac{\beta'}{\alpha}\frac{\hbar\gamma}{2eM}. \quad (16)$$

Continuing this analogy between the energy currents and charge currents, we can generalize the applicability of the result in Eq. (16) to transverse Néel DW (Ref. 6) under the assumption of constant-temperature gradients and vanishing charge currents. Just like the β term is important for the current-driven DWM,[6] the viscous β' term is important for the thermally induced DWM below the Walker breakdown. In a Py wire, a temperature gradient of 1 K/μm should lead to the DW speed of 1 cm/s.

To conclude, we phenomenologically introduced β-like viscous term into the LLGE for the energy currents. We speculate on the possibility of creating heat flows by microwave-induced periodic magnetization dynamics which should result in effective cooling of some regions, in analogy to the Peltier effect. To support it, we considered the DM spiral texture subject to charge current and found that the texture-dynamics-induced heat flow is proportional to the Peltier coefficient and the introduced viscous coupling constant β'. Thus, the materials with large Peltier coefficient, large β' and sufficiently sharp texture should be suitable for the realization of the microwave cooling by magnetization texture dynamics. In some materials, the effective Peltier/Seebeck coefficient as well as the conductivity can be modified and tuned by the texture dynamics and even in situations of pinned textures, the effects of EMF induced by β term should be seen in measurements of the ac conductivity. We also conclude that the β' term is important for the thermally induced DWM. Bauer *et al.*[18] worked out similar ideas for cooling by DWM and thermoelectric excitation of magnetization dynamics.

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[1] M. V. Berry, Proc. R. Soc. London, Ser. A **392**, 45 (1984).
[2] G. E. Volovik, J. Phys.: Condens. Matter **20**, L83 (1987).
[3] S. E. Barnes and S. Maekawa, Phys. Rev. Lett. **98**, 246601 (2007); W. M. Saslow, Phys. Rev. B **76**, 184434 (2007); Y. Tserkovnyak and M. Mecklenburg, *ibid.* **77**, 134407 (2008); R. A. Duine, *ibid.* **77**, 014409 (2008).
[4] Y. Tserkovnyak and C. H. Wong, Phys. Rev. B **79**, 014402 (2009).
[5] S. Zhang and Z. Li, Phys. Rev. Lett. **93**, 127204 (2004); A. Thiaville *et al.*, Europhys. Lett. **69**, 990 (2005); H. Kohno *et al.*, J. Phys. Soc. Jpn. **75**, 113706 (2006); R. A. Duine *et al.*, Phys. Rev. B **75**, 214420 (2007).
[6] Y. Tserkovnyak *et al.*, J. Magn. Magn. Mater. **320**, 1282 (2008).
[7] B. C. Sales, Science **295**, 1248 (2002); A. I. Hochbaum *et al.*, Nature (London) **451**, 163 (2008); A. I. Boukai *et al.*, *ibid.* **451**, 168 (2008).
[8] M. Hatami *et al.*, Phys. Rev. B **79**, 174426 (2009); M. Hatami *et al.*, Phys. Rev. Lett. **99**, 066603 (2007).
[9] L. Berger, J. Appl. Phys. **58**, 450 (1985); S. U. Jen and L. Berger, *ibid.* **59**, 1278 (1986); **59**, 1285 (1986).
[10] H. Ohta *et al.*, Nature Mater. **6**, 129 (2007).
[11] A. Yamaguchi *et al.*, Phys. Rev. Lett. **92**, 077205 (2004); M. Hayashi *et al.*, *ibid.* **96**, 197207 (2006); M. Hayashi *et al.*, Nat. Phys. **3**, 21 (2007).
[12] S. Muhlbauer *et al.*, Science **323**, 915 (2009); F. P. Mena *et al.*, Phys. Rev. B **67**, 241101(R) (2003).
[13] L. Landau and E. Lifshitz, *Electrodynamics of Continuous Media*, 2nd ed. (Pergamon, Oxford, 1984), Vol. 8.
[14] T. Gilbert, IEEE Trans. Magn. **40**, 3443 (2004).
[15] I. D'Amico and G. Vignale, Phys. Rev. B **62**, 4853 (2000).
[16] V. M. Uzdin and A. Vega, Nanotechnology **19**, 315401 (2008).
[17] K. Goto *et al.*, arXiv:0807.2901 (unpublished).
[18] G. E. W. Bauer *et al.* (unpublished).